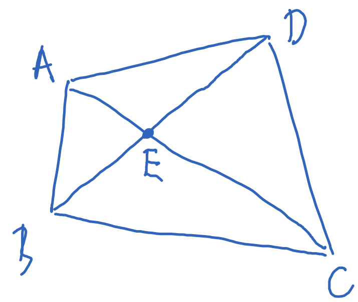
The following example was given in class on Jan 12, 2021.

Ex



A(1, 2, 3)

B(2, 4, 2)

C(0, 1, 1)

D(2, 5, -1)

Find the coordinates of E.

Although one can do this by finding the equation of the lines AC, BD and their intersection, here we use a slightly different method.

Put $a = \overrightarrow{EA}$

$b = \overrightarrow{ED}$

Because A, C, E are on the same line, $a = k\overrightarrow{AC}$

Because B, D, E are on the same line, $b = l\overrightarrow{BD}$.

We have

$$a + \overrightarrow{AD} = \overrightarrow{EA} + \overrightarrow{AD} = \overrightarrow{ED} = b.$$

Thus,

$$k\overrightarrow{AC} + \overrightarrow{AD} = l\overrightarrow{BD}$$

Note that $\vec{AC} = \vec{OC} - \vec{OA} = \langle -1, -1, -2 \rangle$

$$\vec{AD} = \langle 1, 3, -4 \rangle$$

$$\vec{BD} = \langle 0, 1, -3 \rangle$$

Then

$$k\langle -1, -1, -2 \rangle + \langle 1, 3, -4 \rangle = l\langle 0, 1, -3 \rangle$$

$$\begin{cases} -k + 1 = 0 \\ -k + 3 = l \\ -2k - 4 = -3l \end{cases} \rightsquigarrow \begin{cases} k = 1 \\ l = 2 \end{cases}$$

Then $\vec{EA} = a = \vec{AC} = \langle -1, -1, -2 \rangle$.

Let (x, y, z) be the coordinates of E. Then

$$\vec{EA} = \langle 1 - x, 2 - y, 3 - z \rangle.$$

We get

$$\begin{cases} 1 - x = -1 \\ 2 - y = -1 \\ 3 - z = -2 \end{cases} \rightsquigarrow \begin{cases} x = 2 \\ y = 3 \\ z = 5 \end{cases}$$